

In-medium QCD and Cherenkov gluons vs Mach waves at LHC

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The equations of in-medium gluodynamics are proposed. Their classical lowest order solution is explicitly shown for a color charge moving with constant speed. For nuclear permittivity larger than 1 it describes the shock wave induced by emission of Cherenkov gluons. Specific effects at LHC energies are described and compared with Mach wave predictions.

The properties and evolution of the medium formed in ultrarelativistic heavy-ion collisions are widely debated. At the simplest level it is assumed to consist of a set of current quarks and gluons. The collective excitation modes of the medium may, however, play a crucial role. One of the ways to gain more knowledge about the excitation modes is to consider the propagation of relativistic partons through this matter. Phenomenologically their impact would be described by the nuclear permittivity of the matter corresponding to its response to passing partons. Namely this approach is most successful for electrodynamical processes in matter. Therefore, it is reasonable to modify the QCD equations by taking into account collective properties of the quark-gluon medium [1]. Strangely enough, this was not done earlier. For the sake of simplicity we consider here the gluodynamics only.

The classical lowest order solution of these equations coincides with Abelian electrodynamical results up to a trivial color factor. One of the most spectacular of them is Cherenkov radiation and its properties. Now, Cherenkov gluons take the place of Cherenkov photons [2, 3]. Their emission in high-energy hadronic collisions is described by the same formulae but with the nuclear permittivity in place of the usual one. Actually, one considers them as quasiparticles, i.e. quanta of the medium excitations leading to shock waves with properties determined by the permittivity.

Another problem of this approach is related to the notion of the rest system of the medium. It results in some specific features of this effect at LHC energies.

To begin, let us recall the classical in-vacuum Yang-Mills equations

$$D_\mu F^{\mu\nu} = J^\nu, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu], \quad (1)$$

where $A^\mu = iA_a^\mu T_a$; $A_a(A_a^0 \equiv \Phi_a, \mathbf{A}_a)$ are the gauge field (scalar and vector) potentials, the color matrices T_a satisfy the relation $[T_a, T_b] = if_{abc}T_c$, $D_\mu = \partial_\mu - ig[A_\mu, \cdot]$, $J^\nu(\rho, \mathbf{j})$ a classical source current, the metric $g^{\mu\nu} = \text{diag}(+, -, -, -)$.

In the covariant gauge $\partial_\mu A^\mu = 0$ they are written

$$\square A^\mu = J^\mu + ig[A_\nu, \partial^\nu A^\mu + F^{\mu\nu}], \quad (2)$$

where \square is the d'Alembertian operator.

The chromoelectric and chromomagnetic fields are $E^\mu = F^{\mu 0}$, $B^\mu = -\frac{1}{2}\epsilon^{\mu ij}F^{ij}$ or, as functions of the gauge potentials in vector notation,

$$\mathbf{E}_a = -\text{grad}\Phi_a - \frac{\partial \mathbf{A}_a}{\partial t} + gf_{abc}\mathbf{A}_b\Phi_c, \quad \mathbf{B}_a = \text{curl}\mathbf{A}_a - \frac{1}{2}gf_{abc}[\mathbf{A}_b\mathbf{A}_c]. \quad (3)$$

Herefrom, one easily rewrites the in-vacuum equations of motion (1) in vector form. We do not show them explicitly here (see [1]) and write down the equations of the in-medium gluodynamics using the same method as in electrodynamics. We introduce the nuclear permittivity and denote it also by ϵ , since this will not lead to any confusion. After that, one should replace \mathbf{E}_a by $\epsilon\mathbf{E}_a$ and get

$$\epsilon(\text{div}\mathbf{E}_a - gf_{abc}\mathbf{A}_b\mathbf{E}_c) = \rho_a, \quad \text{curl}\mathbf{B}_a - \epsilon\frac{\partial \mathbf{E}_a}{\partial t} - gf_{abc}(\epsilon\Phi_b\mathbf{E}_c + [\mathbf{A}_b\mathbf{B}_c]) = \mathbf{j}_a. \quad (4)$$

The space-time dispersion of ϵ is neglected here.

In terms of potentials these equations are cast in the form

$$\begin{aligned} \Delta \mathbf{A}_a - \epsilon\frac{\partial^2 \mathbf{A}_a}{\partial t^2} = & -\mathbf{j}_a - gf_{abc}\left(\frac{1}{2}\text{curl}[\mathbf{A}_b, \mathbf{A}_c] + \frac{\partial}{\partial t}(\mathbf{A}_b\Phi_c) + [\mathbf{A}_b\text{curl}\mathbf{A}_c] - \right. \\ & \left. \epsilon\Phi_b\frac{\partial \mathbf{A}_c}{\partial t} - \epsilon\Phi_b\text{grad}\Phi_c - \frac{1}{2}gf_{cmn}[\mathbf{A}_b[\mathbf{A}_m\mathbf{A}_n]] + g\epsilon f_{cmn}\Phi_b\mathbf{A}_m\Phi_n\right), \end{aligned} \quad (5)$$

$$\begin{aligned} \Delta \Phi_a - \epsilon\frac{\partial^2 \Phi_a}{\partial t^2} = & -\frac{\rho_a}{\epsilon} + gf_{abc}(2\mathbf{A}_c\text{grad}\Phi_b + \mathbf{A}_b\frac{\partial \mathbf{A}_c}{\partial t} + \frac{\partial \Phi_b}{\partial t}\mathbf{A}_c) - \\ & g^2 f_{amn}f_{nlb}\mathbf{A}_m\mathbf{A}_l\Phi_b. \end{aligned} \quad (6)$$

If the terms with coupling constant g are omitted, one gets the set of Abelian equations, that differ from electrodynamical equations by the color index a only. The external current is due to a parton moving fast relative to partons "at rest".

The crucial distinction between (2) and (5), (6) is that there is no radiation (the field strength is zero in the forward light-cone and no gluons are produced) in the lowest order solution of (2), and it is admitted for (5), (6), because ϵ takes into account the collective response (color polarization) of the nuclear matter.

Cherenkov effects are especially suited for treating them by classical approach to (5), (6). Their unique feature is independence of the coherence of subsequent emissions on the time interval between these processes. The lack of balance of the phase $\Delta\phi$ between emissions with frequency $\omega = k/\sqrt{\epsilon}$ separated by the time interval Δt (or the length $\Delta z = v\Delta t$) is given by

$$\Delta\phi = \omega\Delta t - k\Delta z \cos\theta = k\Delta z\left(\frac{1}{v\sqrt{\epsilon}} - \cos\theta\right) \quad (7)$$

up to terms that vanish for large distances. For Cherenkov effects the angle θ is

$$\cos\theta = \frac{1}{v\sqrt{\epsilon}}. \quad (8)$$

The coherence condition $\Delta\phi = 0$ is valid independent of Δz . This is a crucial property specific for Cherenkov radiation only. The fields (Φ_a, \mathbf{A}_a) and the classical current for in-medium gluodynamics can be represented by the product of the electrodynamical expressions (Φ, \mathbf{A}) and the color matrix T_a .

Let us recall the Abelian solution for the current with velocity \mathbf{v} along z -axis:

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{v}\rho(\mathbf{r}, t) = 4\pi g\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t). \quad (9)$$

In the lowest order the solutions for the scalar and vector potentials are related $\mathbf{A}^{(1)}(\mathbf{r}, t) = \epsilon\mathbf{v}\Phi^{(1)}(\mathbf{r}, t)$ and

$$\Phi^{(1)}(\mathbf{r}, t) = \frac{2g}{\epsilon} \frac{\theta(vt - z - r_\perp\sqrt{\epsilon v^2 - 1})}{\sqrt{(vt - z)^2 - r_\perp^2(\epsilon v^2 - 1)}}. \quad (10)$$

Here $r_\perp = \sqrt{x^2 + y^2}$ is the cylindrical coordinate; z symmetry axis. The cone

$$z = vt - r_\perp\sqrt{\epsilon v^2 - 1} \quad (11)$$

determines the position of the shock wave due to the θ -function in (10). The field is localized within this cone and decreases with time as $1/t$ at any fixed point. The gluons emission is perpendicular to the cone (11) at the Cherenkov angle (8).

Due to the antisymmetry of f_{abc} , the higher order terms (g^3, \dots) are equal to zero for any solution multiplicative in space-time and color as seen from (5), (6).

The expression for the intensity of the radiation is given by the Tamm-Frank formula (up to Casimir operators) that leads to infinity for constant ϵ . The ω -dependence of ϵ (dispersion), its imaginary part (absorption) and chromomagnetic permeability can be taken into account [1].

The attempts to calculate the nuclear permittivity from first principles are not very convincing. It can be obtained from the polarization operator. The corresponding dispersion branches have been computed in the lowest order perturbation theory [5, 6]. The properties of collective excitations have been studied in the framework of the thermal field theories (see, e.g., [7]). The results with an additional phenomenological ad hoc assumption about the role of resonances were used in a simplified model of scalar fields [3] to show that the nuclear permittivity can be larger than 1, i.e. admits Cherenkov gluons. Extensive studies were performed in [8]. No final decision about the nuclear permittivity is yet obtained from these approaches. It must be nontrivial problem because we know that, e.g., the energy dependence of the refractive index of water [9] (especially, its imaginary part) is so complicated that it is not described quantitatively in electrodynamics.

Therefore, we prefer to use the general formulae of the scattering theory to estimate the nuclear permittivity. It is related to the refractive index n of the medium $\epsilon = n^2$ and the latter one is expressed through the real part of the forward scattering amplitude of the refracted quanta $\text{Re}F(0^\circ, E)$ by

$$\text{Re}n(E) = 1 + \Delta n_R = 1 + \frac{6m_\pi^3\nu}{E^2}\text{Re}F(E) = 1 + \frac{3m_\pi^3\nu}{4\pi E}\sigma(E)\rho(E). \quad (12)$$

Here E denotes the energy, ν the number of scatterers within a single nucleon, m_π the pion mass, $\sigma(E)$ the cross section and $\rho(E)$ the ratio of real to imaginary parts of the forward scattering amplitude $F(E)$.

Thus the emission of Cherenkov gluons is possible only for processes with positive $\text{Re}F(E)$ or $\rho(E)$. Unfortunately, we are unable to calculate directly in QCD these characteristics of gluons and have to rely on analogies and our knowledge of the properties of hadrons. The only experimental facts we get for this medium are brought about by particles registered at the final stage. They have some features in common, which (one may hope!) are also relevant for gluons as the carriers of the strong forces. Those are the resonant behavior of amplitudes at rather low energies and the positive real part of the forward scattering amplitudes at very high energies for hadron-hadron and photon-hadron processes as measured from the interference of the Coulomb and hadronic parts of the amplitudes. $\text{Re}F(0^\circ, E)$ is always positive (i.e., $n > 1$) within the low-mass wings of the Breit-Wigner resonances. This shows that the necessary condition for Cherenkov effects $n > 1$ is satisfied at least within these two energy intervals. This fact was used to describe experimental observations at SPS, RHIC and cosmic ray energies. The asymmetry of the ρ -meson shape at SPS [10] and azimuthal correlations of in-medium jets at RHIC [11, 12] were explained by emission of comparatively low-energy Cherenkov gluons [13, 14]. The parton density and intensity of the radiation were estimated. In its turn, cosmic ray data [15] at energies corresponding to LHC require very high-energy gluons to be emitted by the ultrarelativistic partons moving along the collision axis [2]. Let us note the important difference from electrodynamics, where $n < 1$ at high frequencies.

The in-medium equations are not Lorentz-invariant. There is no problem in macroscopic electrodynamics, because the rest system of the macroscopic matter is well defined and its permittivity is considered there. For collisions of two nuclei (or hadrons) it requires special discussion.

Let us consider a particular parton that radiates in the nuclear matter. It would "feel" the surrounding medium at rest if the momenta of all other partons, with which this parton can interact, are smaller and sum to zero. In RHIC experiments the triggers, that registered the jets (created by partons), were positioned at 90° to the collision axis. Such partons should be produced by two initial forward-backward moving partons scattered at 90° . The total momentum of the other partons (medium spectators) is balanced, because for such a geometry the partons from both nuclei play the role of spectators forming the medium. Thus the center of mass system is the proper one to consider the nuclear matter at rest in this experiment. The permittivity must be defined there. The Cherenkov rings consisting of hadrons have been registered around the away-side jet, which traversed the nuclear medium. This geometry requires, however, high statistics, because the rare process of scattering at 90° has been chosen.

The forward (backward) moving partons are much more numerous and have higher energies. However, one cannot treat the radiation of such a primary parton

in the c.m.s. in a similar way, because the momentum of the spectators is different from zero, i.e. the matter is not at rest. Now the spectators (the medium) are formed from the partons of another nucleus only. Then the rest system of the medium coincides with the rest system of that nucleus and the permittivity should refer to this system. The Cherenkov radiation of such highly energetic partons must be considered there. That is what was done for interpretation of the cosmic ray event in [2]. This discussion shows that one must carefully define the rest system for other geometries of the experiment with triggers positioned at different angles.

Thus our conclusion is that the definition of ϵ depends on the geometry of the experiment. Its corollary is that partons moving in different directions with different energies can "feel" different states of matter in the **same** collision of two nuclei because of the permittivity dispersion. The transversely scattered partons with comparatively low energies can analyze the matter with rather large permittivity corresponding to the resonance region, while the forward moving partons with high energies would "observe" a low permittivity in the same collision. This peculiar feature can help scan the $(\ln x, Q^2)$ -plane as discussed in [16]. It explains also the different values of ϵ needed for the description of the RHIC and cosmic ray data.

These conclusions can be checked at LHC, because both RHIC and cosmic ray geometry will become available there. The energy of the forward moving partons would exceed the thresholds above which $n > 1$. Then both types of experiments can be done, i.e. the 90°-trigger and non-trigger forward-backward partons experiments. The predicted results for 90°-trigger geometry are similar to those at RHIC. The non-trigger Cherenkov gluons should be emitted within the rings at polar angles of tens degrees in c.m.s. at LHC by the forward moving partons (and symmetrically by the backward ones) according to some events observed in cosmic rays [15, 14].

Let us compare the conclusions for Cherenkov and Mach shock waves. The Cherenkov gluons are described as the transverse waves while the Mach waves are longitudinal. Up to now, no experimental signatures of these features were proposed.

The most important experimental fact is the position of the maxima of humps in two-particle correlations. They are displaced from the away-side jet by 1.05-1.23 radian [17, 18, 19]. This requires rather large values of $\text{Re}\epsilon \sim 2 - 3$ and indicates high density of the medium [14] that agrees with other conclusions. The fits of the humps with complex permittivity are in progress. The maxima due to Mach shock waves should be shifted by the smaller value 0.955 if the relativistic equation of state is used ($\cos \theta = 1/\sqrt{3}$). To fit experimental values one must consider different equation of state. In three-particle correlations, this displacement is about 1.38 [11].

There are some claims [11, 12] that Cherenkov effect contradicts to experimental observations because it predicts the shift of these maxima to smaller angles for larger momenta. They refer to the prediction made in [3]. However, the conclusions of this paper about the momentum dependence of the refractive index can hardly be considered as quantitative ones because the oversimplified scalar Φ^3 -model with simplest resonance insertions was used for computing the refractive index. In view

of difficult task of its calculation discussed above, the fits of maxima seem to be more important for our conclusions about the validity of the two schemes.

Mach waves should appear for forward moving partons at RHIC but were not found. The energy threshold of ϵ explains this phenomenon for Cherenkov gluons.

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